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Computational modeling of a generalized Buckley-Leverett equation with diffusive discontinuous capillary pressure

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In recent works, several approaches for treating spatially discontinuous capillary pressure models in multiphase flow in porous media [2, 3]. Hybridized mixed finite elements formulations for parabolic and elliptic problems have been succesfuly applied in multiphase flow problems [2], and were identified as very well suited for the accurate calculation of gradient flow associated with velocity fields (Darcy's law) and diffusive fluxes in the presence of high contrast geological properties in permeability and porosity fields. It is worth mentioning that the finite element method in the mixed hybrid form possesses rigorous mathematical foundation, delicately connected to distinct approaches in numerical analysis for elliptical problems. The work [2] presented a combination of a conservative central scheme to handle a system of nonlinear conservation laws modeling the convective transport of the fluid phases and locally conservative mixed finite elements for the associated parabolic and elliptic problems in an operator splitting manner. Such algorithm for approximating the solution of three-phase, two-dimensional immiscible flow in porous media takes into account gravitational effects and explicit spatially varying capillary pressure models and discontinuous flux functions. However, the spatial dependence of the capillary pressure models studied in [2] is multiplicative, based on a Leverett-like J-function:

$$P_c(S, K(x)) = p_c(S)\sqrt{\frac{K(x)}{\phi(x)}}, \tag{1}$$

where p_c incorporates the dependency on S and the remaining term incorporates spatial dependency. However, the situation exhibits other challenges on capturing the correct entropy solution with nonclassical shocks arising in more general models of capillary pressure, such as the additive capillary pressure model from [3]:

$$P_c(S, K(x)) = P_{L,R} - \log(1 - S), \tag{2}$$

where the term $P_{L,R}$ incorporates the spatial dependency of the model (for $x < 0$ we must have P_L and for $x > 0$ we must have P_R). The work [3] handles the identification of the correct entropy solution by giving a procedure that determines the appropriate connection in terms of the flux profiles and capillary pressure profiles. The procedure consists on building a finite volume numerical method for the Buckley-Leverett equation with an interface coupling that retains information from the vanishing capillarity model. This formulation, though, is challenged by the difficulty of extending the two-phase scalar problem to the three-phase system problem.

The works [1, 4, 5] made use of a fully-coupled finite volume/hybrid mixed finite element formulation on the numerical approximation of several nontrivial transport models. In order to overcome the drawbacks from approaches [2, 3], we present a unified hybrid mixed finite element and finite volume formalism along a novel reinterpretation of Robin coupling conditions for numerically solving convection-diffusion problems with gravity and diffusive discontinuous capillary pressure [6]. This reinterpretation can be seen as a generalization of the Rankine-Hugoniot jump conditions. make use of the finite element method in a hybridized mixed formulation with the Raviart-Thomas approximating spaces for the diffusive transport, proposing a reinterpretation of the interface conditions between elements to accommodate the effects of heterogeneities in the diffusive capillary pressure. The hybrid mixed finite element method with this novel interface condition is then coupled with finite volume methods in a conservative form for approximating the convective transport. The convective operator is numerically approached as a source term in the hybrid mixed finite element formulation in order to preserve the delicate nonlinear balance with the diffusive operator. We also present some preliminary numerical tests within this formulation and this is currently a work in progress.

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