



Stochastic continuity equation with non-smooth velocity

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In this work we study the following stochastic linear transport/continuity equation

$$\begin{cases} \partial_t u(t, x) + \text{Div}\left(\left(b(t, x) + \frac{dB_t}{dt}\right) \cdot u(t, x)\right) = 0, \\ u|_{t=0} = u_0, \end{cases} \quad (1)$$

where $(t, x) \in [0, T] \times \mathbb{R}^d$, $\omega \in \Omega$ is an element of the probability space $(\Omega, \mathcal{F}, \mathbb{P})$, $b : \mathbb{R}_+ \times \mathbb{R}^d \rightarrow \mathbb{R}^d$ is a given vector field and $B_t = (B_t^1, \dots, B_t^d)$ is a standard Brownian motion in \mathbb{R}^d . The stochastic integration is to be understood in the Stratonovich sense. A very interesting situation is when the stochastic problem is better behaved than the deterministic one. A first result in this direction was given by F. Flandoli, M. Gubinelli and E. Priola in [1], where they obtained well-posedness of the stochastic problem (1) for an Hölder continuous drift term, with some integrability conditions on the divergence. Their approach is based on a careful analysis of the characteristics.

Só, the main issue of this work is to prove uniqueness of L^2 -weak solutions for one-dimensional stochastic continuity equation (1) with unbounded measurable drift without assumptions on the divergence. More precisely, we assume that b satisfies

$$|b(x)| \leq k(1 + |x|).$$

The proof is based in the fact that one primitive V is regular and it verifies the transport equation

$$\partial_t V(t, x) + \left(b(t, x) + \frac{dB_t}{dt}\right) \cdot \nabla V(t, x) = 0. \quad (2)$$

Then using a modified version of the “commutator Lemma” and the characteristic systems associated to the stochastic partial differential equation (2) we shall show that $V = 0$ with initial condition equal to zero, which implies that $u = 0$. For more details of this result to see [2]. Joint work with Christian Olivera (Universidade Estadual de Campinas).

Referências

- [1] F. Flandoli, M. Gubinelli and E. Priola, *Well-posedness of the transport equation by stochastic perturbation*, *Invent. Math.*, 180, 1-53 pages, 2010.
- [2] D. A. C. Mollinedo and C. Olivera, *Stochastic continuity equation with non-smooth velocity*, *Annali di Matematica Pura ed Applicata (1923 -)*, page 1-16, 2017.

Palavras-chave: *Stochastic partial differential equation, Continuity Equation, Stochastic characteristic method, Regularization by noise, Commutator Lemma.*