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A Multiscale Approach to the Asymptotic Behavior of Solutions to Nonlinear Integral Equations with Generalized Heat Kernel

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Here we study a class of nonlinear integral equations with a generalized heat kernel, obtaining global existence and uniqueness of the solution, as well as the asymptotic behavior, using the Renormalization Group method. The nonlinearities are classified according to their contribution to the asymptotic behavior. We show that the so called *irrelevant* perturbations do not affect the asymptotic profile of the solution, in the sense that the profile is the same as in the linear case, whereas, by adding *marginal* perturbations, an extra logarithmic factor appears on the decay of the solution.

More specifically, we obtain the asymptotic behavior of solutions to equations of type

$$u(x,t) = \int G(x-y,s(t))f(y)dy + \int_1^t \int G(x-y,s(t)-s(\tau))F(u(y,\tau))dyd\tau, \tag{1}$$

with $x \in \mathbb{R}$ and $t > 1$. By imposing conditions on the kernel without specifying $G = G(x,t)$, we generalize the study of asymptotics for initial value problems. This outlook was adopted in [1, 2] where it is shown that, under certain conditions on G, with $s(t) = t$, the solution $u(x,t)$ to (1) behaves for long time as

$$\frac{A}{t^{1/d}} G\left(\frac{x}{t^{1/d}}, 1\right),$$

where $d > 0$ is such that $G(x,t) = t^{-\frac{1}{d}} G\left(t^{-\frac{1}{d}}x, 1\right)$. We recover and extend the above result using a multiscale approach showing that, if $c(t)$ is a positive function in $L^1_{loc}((1,+\infty))$ of type $t^p + \mathrm{o}(t^p)$, with $p > 0$ and

$$s(t) = \int_1^t c(\tau)d\tau = \frac{t^{p+1}-1}{p+1} + r(t),$$

then, for $F(u) = \sum_{j\geq\alpha} a_j u^j$ with $\alpha > (p+1+d)/(p+1)$,

$$u(x,t) \sim \frac{A}{t^{(p+1)/d}} G\left(\frac{x}{t^{(p+1)/d}}, \frac{1}{p+1}\right) \text{ when } t \to \infty.$$

Furthermore, if $F(u) = -\lambda u^\alpha + \sum_{j>\alpha} a_j u^j$ with λ small and positive and $\alpha = (d+p+1)/(p+1)$, then

$$u(x,t) \sim \frac{A}{(t\ln t)^{(p+1)/d}} G\left(\frac{x}{t^{(p+1)/d}}, \frac{1}{p+1}\right) \text{ when } t \to \infty.$$

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