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Pullback Attractors for Multivalued Processes and Application to Nonautonomous Problems with Dynamic Boundary Conditions

Rodrigo A. Samprogna, **Karina Schiabel**, **Claudia B. Gentile Moussa**,
DM - Departamento de matemática - UFSCar - SP
Caixa Postal 676, 13.565-905 São Carlos SP, Brazil
E-mail: samprogna@hotmail.com, schiabel@dm.ufscar.br, gentile@dm.ufscar.br.

In this work we prove, by using the Faedo-Galerkin method, the existence of at least one weak solution for the following nonautonomous p-Laplacian problem,

$$\begin{cases} u_t - \Delta_p u + f_1(t,u) = g_1(t,x), & (t,x) \in (\tau,+\infty)\times\Omega, \\ u_t + |\nabla u|^{p-2}\partial_{\overrightarrow{n}} u + f_2(t,u) = g_2(t,x), & (t,x) \in (\tau,+\infty)\times\Gamma, \\ u(\tau) = u_0, \end{cases} \tag{P}$$

where Ω is a bounded domain of $\mathbb{R}^N$ $(N \geq 3)$ with smooth boundary $\partial\Omega = \Gamma$, Δ_p denotes the p-Laplacian operator, defined by $\Delta_p u = div(|\nabla u|^{p-2}\nabla u)$, $\overrightarrow{n}$ is the outer normal to Γ, u_0 is an initial state in a suitable space, and $p \in [2,+\infty)$. The perturbations f_i and the external forces g_i, $i = 1,2$, satisfy the following assumptions:

(H1) $g_1 \in L^{p'}_{loc}(\mathbb{R}; L^{r_1'}(\Omega))$, $g_2 \in L^{p'}_{loc}(\mathbb{R}; L^{r_2'}(\Gamma))$ where p' denotes the conjugate exponent of p, with

$$r_1 \in \begin{cases} (p, pN/(N-p)], & \text{if } p \in [2,N), \\ (p,+\infty), & \text{if } p = N, \\ [p,+\infty), & \text{if } p > N \end{cases} \tag{1}$$

and

$$r_2 \in \begin{cases} [2,(N-1)p/(N-p)], & \text{if } p \in [2,N), \\ [2,+\infty), & \text{if } p \geq N; \end{cases} \tag{2}$$

(H2) $f_i \in C(\mathbb{R}^2)$ satisfies

$$a_i(t)|s|^{r_i} - k_i(t) \leq f_i(t,s)s, \tag{3}$$

a.e for $t \in \mathbb{R}$ and every $s \in \mathbb{R}$ with $a_i \in L^1_{loc}(\mathbb{R})$ real functions such that $a_i(t) \geq a_0 > 0$ for some fixed real number a_0, and $k_i \in L^1_{loc}(\mathbb{R})$ are positive functions, for $i = 1,2$;

(H3) There are functions $C_i \in L^\infty_{loc}(\mathbb{R})$, $i = 1,2$, such that $|f_i(t,s)| \leq C_i(t)(|s|^{r_i-1}+1)$ a.e. for $t \in \mathbb{R}$ and each $s \in \mathbb{R}$.

A autonomous version of this problem can be found in [5] and with a nonnautonomous term in external forces g_i can be found in [6, 9].

We also extend to the multivalued context results on the existence of $\mathscr{D}$-pullback attractor, a pullback attractor for a determined basin of attraction $\mathscr{D}$, see [2, 3] for well posed problems. We chose to use the generalized process approach to work in the multivalued context, this approach allows us to impose suitable conditions on the curves rather than require only the desired properties on each time state, [1, 4, 8]. We prove the existence of such attractors for the class of nonautonomous problems we are concerned about.

The results of this research can be found in the work [7].

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